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3.3.2 Number of books and chapters in edited volumes/books published and papers published in national/ international conference proceedings per teacher during last five year

Sl. No.	Name of the teacher	Title of the book/chapters published	Title of the paper	Title of the proceedings of the conference	Name of the conference	National / International	Year of publication	ISBN number of the proceeding	Affiliating Institute at the time of publication	Name of the publisher
1	Dr. S.K. Malhotra	A Common Fixed Point Theorem In Fuzzy Cone Metric Spaces	A Common Fixed Point Theorem In Fuzzy Cone Metric Spaces	Proceedings National Conference-Recent Development in Mathematical Modeling and Fuzzology second	National Conference-Recent Development in Mathematical Modeling and Fuzzology second	National	2017-18	976-93-5261-070	Government Dr. Shyama Prasad Mukherjee Science and Commerce College Bhopal	Chandigarh University
2	Dr. S.K. Malhotra	Cyclic Chaterjea Type Mappings and Fixed Point Theorems in cone metric spaces	Cyclic Chaterjea Type Mappings and Fixed Point Theorems in cone metric spaces	Proceedings National Conference-Recent Development in Mathematical Modeling and Fuzzology second	National Conference-Recent Development in Mathematical Modeling and Fuzzology second	National	2017-18	976-93-5261-070	Government Dr. Shyama Prasad Mukherjee Science and Commerce College Bhopal	Chandigarh University
3	Dr. S.K. Malhotra	Cyclic Chaterjea Type Mappings and Fixed Point Theorems in cone metric spaces	Cyclic Chaterjea Type Mappings and Fixed Point Theorems in cone metric spaces	Proceedings National Conference- Recent Advancement In Developing India	Proceedings National Conference- Recent Advancement In Developing India	National	2017-18	978-93-87738-11-08	Government Dr. Shyama Prasad Mukherjee Science and Commerce College Bhopal	Springer
4	Dr. S.K. Malhotra	Fixed Point Theorems For Ordered Expanding mappings On Ordered Cone Metric Spaces Over Banach Algebras	Fixed Point Theorems For Ordered Expanding mappings On Ordered Cone Metric Spaces Over Banach Algebras	Multi-Disciplinary National Conference on Recent Adavancement in Developning India	Multi-Disciplinary National Conference on Recent Adavancement in Developning India	National	2017-18	978-93-87738-11-8	Government Dr. Shyama Prasad Mukherjee Science and Commerce College Bhopal	Springer
5	Dr. Anita Mandloi	A Study of the Regularity of Two Interpolation Matrices	A Study of the Regularity of Two Interpolation Matrices	Multi-Disciplinary National Conference on Recent Adavancement in Developning India	Multi-Disciplinary National Conference on Recent Adavancement in Developning India	National	2017-18	978-93-87738-11-08	Government Dr. Shyama Prasad Mukherjee Science and Commerce College Bhopal	Springer
6	Dr. Sudhanshudhar Dwivedi	B.Sc. I year Part2	Inorganic Chemistry		Review and Editing of Books	National	2017-18	978-93-51400-89-9	Government Dr. Shyama Prasad Mukherjee Science and Commerce College Bhopal	Pragati Prakashan Merrut

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Cyclic Chatterjea type mappings and fixed theorems in cone metric spaces

S.K. Malhotra, P.K. Bhargav, Satish Shukla

Abstract— The purpose of this paper is to introduce the cyclic mappings in cone metric spaces over Banach algebra and to prove for such mappings. An example is presented which illustrate the [illegible]

Index Terms—Cyclic mapping; Chatterjea type mapping, cyclic type metric space, fixed point; Banach algebra, contraction principle.

1 Introduction

In 1922, Banach proved the famous Banach contraction principle. It says that:

Theorem 1.1 Let (X, d) be a complete metric space and $T: X \to X$ be a mapping satisfying the following condition: there exists $\lambda \in [0,1)$ such that

$$d(Tx, Ty) \leq \lambda d(x, y) \ \forall\ x, y \in X. \quad (1)$$

Then T has a unique fixed point in X.

The Banach contraction principle have several generalization and applications in pure and applied mathematics. A mapping satisfying condition (1) is called Banach contraction. It is easy to see that every Banach contraction is a continuous mapping. Indeed, it is uniform continuous mapping. Therefore, in the Banach contraction principle the mapping T is necessarily continuous. But in particle problems the mappings under consideration may be discontinuous. It was a major drawback of Banach contraction principle that the mappings having fixed point must be continuous.

Kannan [8,9] first introduced the following contractive condition: there exists $\lambda \in [0,1/2)$ such that

$$d(Tx, Ty) \leq \lambda[d(x, Tx) + d(y, Ty)]$$

Kannan [8,9] [illegible] satisfying condition (2) [illegible] space has a unique fixed point [illegible] satisfying condition (2) is called [illegible] mapping. It is shown in [8,9] [illegible] (1) and (2) are independent [illegible] that a Kannan type mapping [illegible] continuous. Thus, Kannan [illegible] necessity of continuity [illegible] existence of fixed point.

Chatterjea [10] considered the following condition:

there exists $\lambda \in [0,1/2)$ such that

$$d(Tx, Ty) \leq \lambda[d(x, Ty) + d(y, Tx)] \quad \forall\ x, y \in X.$$

A mapping satisfying condition [illegible] Chatterjea type mapping. Chatterjea [illegible] the a Chatterjea type mapping [illegible] space has a unique fixed point [illegible] Chatterjea type mappings is also [illegible] drawback of continuity.

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FIXED POINT THEOREMS FOR ORDERED EXPANDINGMAPPINGS ON ORDERED CONE METRIC SPACES OVER BANACH ALGEBRAS

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ABSTRACT: In this paper, we prove some fixed point theorems for an orderedexpanding mapping on ordered cone metric spaces over Banach algebras.Results of this paper generalize and extend the results of Liu and Xu [4] and-Ran and Reurings [1] and and Nieto and R.R. Lopez [6,7] in cone metric spaces over Banach algebras. An exampleis provided which illustrates the results proved herein.

Keywords: Ordered cone metric space; fixed point; expanding mapping; partial ordered set.

MSC 2000: 54H25; 47H10.

INTRODUCTION

Huang and Zhang [9] defined the cone metric spaces and proved some fixed point theorems for various type of contractive mappings and generalize the famous Banach contraction principle [12] in various ways. Some recent studies (see, [2,19]) show that the fixed point results proved in conemetric space are direct consequences of their usual metric versions. To overcomethis drawback, recently, Liu and Xu [4] improved the concept of cone metric spacesby defining the cone metric into Banach algebras, instead of a Banach space, so that, incontractive conditions they could use a vector instead of a constant scalar. The fixed pointresults thus obtained were free from this drawback which was shown by an examplein [4].Ran and Reurings [1] obtained the existence of fixed points of a self mapping of a metric space equipped with a partial order. In this article, an extension of the results ofLiu and Xu [4], Ran and Reurings [1]and Nieto and R.R. Lopez [6,7],has been worked out by presenting a fixed point result for anordered expanding mapping on a cone metric space over Banach algebra equipped with a partial order.An exampleis provided which illustrate the results proved herein.

PRILIMINARIES

Throughout this paper $\mathbb{R}$ and $\mathbb{N}$ will denote the set of all real and the set of all natural numbers respectively.

Let A be a real Banach algebra, i.e., A is a real Banach space in which an operation of multiplication is defined, subject to the following properties (see, [18]): for all $x, y, z \in A; a \in \mathbb{R}$ (i) $x(yz) = (xy)z$; (ii) $x(y+z) = xy + xz$ and $(x+y)z = xz + yz$; (iii) $a(xy) = (ax)y = x(ay)$; (iv) $\| xy \| \leq \| x \| \| y \|$.

In this paper, we shall assume that the Banach algebra A has a unit, i.e., a multiplicative identity e such that $ex = xe = x$ for all $x \in A$. An element $x \in A$ is said to be invertible if there is an inverse element $y \in A$ such that $xy = yx = e$. The inverse of $x \in X$ is denoted by x^{-1}. For more details we refer to [18].

Proposition2.1. Let A be a real Banach algebra with the unit e and $x \in A$. If the spectral radius $\rho(x)$ of x is less than one, i.e.,$\rho(x) = \lim_{n\to\infty} \|x^n\|^{\frac{1}{n}} = \inf_{n\geq 1} \|x^n\|^{\frac{1}{n}} < 1$.

Then, $e - x$ is invertible. Actually,$(e-x)^{-1} = \sum_{i=0}^{\infty} x^i$.

A subset P of A is called a cone if:

1. P is closed and $\{\theta, e\} \subset P$, where θ is the zero vector of A;
2. $\alpha P + \beta P \subset P$ for all non-negative real numbers α, β;
3. $P^2 = PP \subset P$;
4. $P \cap (-P) = \{\theta\}$.

For a given cone $P \subset A$, we can define a partial ordering $\preccurlyeq$ in A with respect to P by $x \preccurlyeq y$ (or equivalently $y \succcurlyeq x$) if and only if $y - x \in P$. The notation $x \ll y$ (or equivalently $y \gg x$) will stand for $y - x \in P^{\circ}$, where P°denotes the interior of P.The cone P is called normal if there exists a number $K > 0$ such that for all $x, y \in P, x \preccurlyeq y$ implies$\|x\| \leq K\|y\|$.The least positive value of K satisfying the above inequality is called the normal constant of P (see [17]). In the following we always assume that P is a cone in a real Banach algebra A with $P^{\circ} \neq \emptyset$ (i.e., the cone P is a solid cone) and $\preccurlyeq$ is the partial ordering with respect to P.

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A STUDY OF THE REGULARITY OF TWO INTERPOLATION MATRICES

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Anita Mandloi

Govt. Science & Commerce College Benazeer Bhopal (M.P.)

ABSTRACT: We study a theorem which connects the regularity of two interpolation matrices. An Interpolation scheme

$$P_n^k(xi) = C_{i,k} \quad i=1,2,\ldots,m^* \quad k = 0, 1,2 ,\ldots,n \qquad \textbf{(A)}$$

is generally described an m x (n+1) interpolation matrix given by

$$E = (e_{i,k}) \qquad i = 1,2\ldots m , k = 0,1,2,\ldots n.$$

with $e_{i,k} = 1$ if condition (A) is true otherwise $e_{i,k} = 0$ clearly $e_{i,k} = 1$ indicates that k-th derivative of the polynomial P_n at xi is being interpolated.

An interpolation matrix is called normal if the number of i's in E is equal to the number of columns i.e. n+1.

Keywords: Set of nodes, fundamental polynomials, set of polynomials of degree at most n, polya condition, regularity, annihilated function.

INTRODUCTION:

Let Π_n denotes the set of polynomials of degree at most n. Then given a set of discrete data ck,l a set of distinct nodes x1,x2,.....xr with multiplicities mk, k = 1,2..... r such that Σmk = n + 1, the basic interpolation problem is to determine a polynomial Pn ∈ Π n, such that the system of n + 1 equations

$$P^l n\,(xk) = ck,l \text{ holds} \qquad \textbf{(1.1)}$$

We use some basic definitions related to regularity of interpolation matrices.

Set of nodes The set X = {x1,x2,..... .xm} of nodes is the set of all those points which are included in an interpolation scheme given by equations of the type (1.1).

Generating functions Real valued functions $g_0 g_1 \ldots\ldots, g_n$ defined on an interval [a,b] and forming a basis in C^n to generate the interpolating polynomial are called the generating functions. Hence, if Pn is the interpolating polynomial then

$$Pn = \Sigma\, a_j g_j \qquad j=0,1,2\ldots,n \qquad \textbf{(1.2)}$$

Algebraic interpolation If the generating functions are given by

$$gj(x) = x^j/j! \qquad 0 \le j \le n \qquad \textbf{(1.3)}$$

then the interpolating polynomial Pn becomes an algebraic polynomial and the scheme of interpolation is known as algebraic interpolation.

Interpolation matrix An interpolation scheme

$$P_n^{(k)}\,(xi) = ei;k;\ i = 1,2,\ldots\ldots m,\ k = 0,1,\ldots n \qquad \textbf{(1.4)}$$

is generally described by an mx(n+1) interpolation matrix given by

$$E = (ei;k) \qquad i=1,2,\ldots m \quad k=0,1,\ldots n \qquad \textbf{(1.5)}$$

with ei;k = 1 if condition (1.4) is true otherwise ei;k = 0.

Clearly, ei;k = 1 indicates that k-th derivative of the polynomial Pn at xi is being interpolated.

Normal interpolation matrix An interpolation matrix is called normal if the number of 1's in E is equal to the number of columns i.e. n + 1. Conventionally, we write

$$|E| = \Sigma ei,k = n + 1, \qquad i = 1,2,\ldots\ldots m,\ k = 0,1,\ldots\ldots n$$

Let G = {g0,g1,.....gn} is the class of generating functions so that

$$Pn = \Sigma a_k g_k \quad k=0,1,\ldots\ldots n$$

and the determinant of the coefficient matrix of the system is given by

$$D(E,X) := D(E,X;G) := D(E,X;\, g0,g1,\ldots\ldots gn) \qquad \textbf{(1.6)}$$

$$:= \det\,[g_0^{(k)}(xi),\ g_1^{(k)}(xi)\ldots\ldots g_n^{(k)}(xi) : ei,k = 1;\ ei,k \in E] \qquad \textbf{(1.7)}$$

If D(E,X) ≠ 0, we have a unique solution of the system (1.4) by Cramer's rule

$$Pn(x) = \Sigma\, D(E,X,G_k) / D(E,X,G) \times gk(x) \quad k=0,\ldots\ldots n \qquad \textbf{(1.8)}$$

where the system Gk consists of g0,.....gk-1,f,gk+1,.....gn: The A(E,X) will denote the corresponding matrix to D(E,X), it means

$$A(E,X) = [g_0^{(k)}(xi),\ g_1^{(k)}(xi),\ldots\ldots g_n^{(k)}(xi)\ ,ei,k = 1,\ ei,k \in E].$$

Annihilated function A function Pn is called annihilated by the pair (E,X) if it satisfies the homogeneous system corresponding to the scheme (1.4), which means

$$P_n^{(k)}(xi) = 0;\ ei,k = 1;\ ei,k \in E. \qquad \textbf{(1.9)}$$

We define the class of annihilated functions as

$$A(E,X,G) = \{Pn = \Sigma\, ak gk : P_n^{(k)}(xi) = 0\ \ ei,k = 1;\ ei,k \in E\}\ \ k=0,1,\ldots\ldots n \qquad \textbf{(1.10)}$$

Managing an explicit representation

Let us assume that the pair (E,X) is regular. We call Aik ∈ Π̇n the fundamental polynomials for the pair (E,X), where with kronecker delta notation

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$$A_{ik}^{(l)}(x_q) = \delta_{kl}\delta_{iq}, \quad e_{i,k} = e_{q,l} = 1,\ e_{i,k}, e_{q,l} \in E. \qquad (1.11)$$

Then Pn in the interpolation scheme (1.4) is given by

$$P_n(x) = \Sigma_{e_{i,k}=1, e_{i,k}\in E} c_{i,k} A_{ik}(x) \qquad (1.12)$$

NOW I PRESENT A THEOREM WHICH CONNECTS THE REGULARITY OF TWO POLATION MATRICES:

Theorem Let mx(n+1) normal interpolation matrices E and E' be of the forms

$$E = E1 + E2,\ E' = E1 + E'2,\ |E2| = |E'_2| = s\ (1 \le s \le n), \qquad (2.1)$$

Assume that for a set of nodes X the pair (E',X) is regular. Then the following statements are equi

(a) The pair (E,X) is regular.

(b) There exist s fundamental polynomials Aik $\in \Pi n$, ei,k = 1, ei,k $\in$ E2, for the pair (E,X).

(c) There exist s linearly independent functions hi $\in \Pi n$, i = 0,1,.....,s-1, annihilated by E1,) the determinant D(E2, X; h0,.....hs-1)

= det [$h_0^{(k)}$ (xi),..... $h_{s-1}^{(k)}$ (xi) ; $e_{i,k}$ = 1; $e_{i,k}$ $\in$ E2] . (2.2)

is nonzero.

Proof (a) =>(b) is trivial. Next follows the proof of (b) => (c).

We order the s polynomials A_{ik}, $e_{i,k}$ = 1, $e_{i,k}$ $\in$ E2 lexicographically and denote h0,..... hs-1, If we order the pair in (2.2) in the same order then by (1.11) the matrix that (2.2) is the unit matrix which implies that the determinant (2.2) is nonzero.

For proving that (c) =>(a), let fi,k $\in \Pi n$, e'I,k = 1, e'I,k $\in$ E be the fundamental polynomials ((E',X) Then the fi,k must exist uniquely, since the pair (E',X) is regular.

Meanwhile, the fi,k span the set Πn. We rewrite the polynomials

Fi,k $\in \Pi n$; e'I,k = 1; e'I,k $\in$ E'2 as g0,....,gs-1 in their lexicographical order and fi,k $\in \Pi n$; e'I,k = E'1 as gs,.....gn in their lexicographical order, too. Thus the pair (E,X) is regular if and determinant D(E,X; g0; g1,.....gn) is nonzero. We rearrange the corresponding matrix (g1,.....gn) to have as the first s rows those corresponding to ei,k=1 from E2 in their lexic order, then rows corresponding to ei,k = 1 from E1 in their lexicographical order also. This transf A(E,X; g0,g1,.....gn) into

$$\begin{bmatrix} A(E2,X;\ g0,\ g1,\ldots\ldots gs-1) & \\ 0 & A(E1,X;\ gs,\ldots\ldots gn) \end{bmatrix} \qquad (2.3)$$

Hence

$$D(E,X;\ g0,\ g1,\ldots\ldots gn)$$
$$= D(E2,X;\ g0,\ldots\ldots gs-1)D(E1,X;\ gs,\ldots\ldots gn) \qquad (2.4)$$

According to (11) we have that A(E1,X; gs,.....gn) is the unit matrix and hence

$$D(E1,X;\ gs,\ldots\ldots gn) = 1,$$

Then

$$D(E,X;\ g0,g1,\ldots\ldots gn) = D(E2,X;\ g0,\ldots\ldots gs-1) \qquad (2.5)$$

Thus the proof follows if we show that with a nonzero constant C

$$D(E2,X;\ h0,\ldots\ldots hs-1) = CD(E2,X;\ g0,\ldots\ldots gs-1). \qquad (2.6)$$

In fact, since h0,.....,hs-1 are annihilated by the pair (E1,X), by the definitions of g0, gs,.....,gn we obtain

$$h_a = \Sigma c_{aj} g_j, \quad a = 0,1,2,\ldots\ldots s-1,\ j = 0,1,\ldots\ldots s-1 \qquad (2.7)$$

Which implies

$$h_a^{(k)}(xi) = \Sigma\, c_{aj}\, g_j^{(k)}(xi), \quad a = 0,1,\ldots\ldots s-1,\ ei,k = 1,\ ei,k \in E2.$$
$$j = 0,1,\ldots\ldots s-1$$

Thus

A(E2,X,h0 hs-1)

= $[c_{aj},\ a, j = 0, 1, \ldots, s-1]$x A(E2,X; g0,.... gs-1)^2

This yield

D(E2,X; h0,....,hs-1)

= det$[c_{aj},\ a, j = 0,1,\ldots\ldots s-1]$ X D(E2,X; g0,.....gs-1) Since h0, ... hs-1 are linearly independe ; a, j = 0, 1,..... s - 1] $\neq$ 0 **which proves (2.6).**

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