Lecture on Boolean Algebra and its properties

Present by

Dr Naval Singh

Professor

Department of Mathematics

Govt. Dr Shyama Prasad Mukharjee Science and Commerce college

Bhopal M.P.

Definition: A complemented, distributive lattice is called boolean algebra

Theorem 1: In Boolean algebra, prove that

(i) Additive identity if exists is unique

(ii) Multiplicative identity if exists is unique

(iii) The complement of each element if exists is unique

Proof : let B be a boolean algebra

(i) Let O_1 and O_2 be two additive identities

taking O_1 as identity $O_1 + O_2 = O_2$ ...(1)

taking O_2 as identity $O_1 + O_2 = O_1$ ...(2)

from equation (1) and (2) $O_1 = O_2$

Hence additive identity is unique

(ii) Let I_1 and I_2 be two multiplicative identities

taking I_1 as identity $I_1 \cdot I_2 = I_2$ ...(1)

taking I_2 as identity $I_1 \cdot I_2 = I_1$ ...(2)

from equation (1) and (2) $I_1 = I_2$

Hence multiplicative identity is unique

(iii)Let b and c be two compliments of a,

Then $a + b = 1,\ a + c = 1,\ a \cdot b = 0,\ a \cdot c = 0$

Now $b = b + 0$

$= b + a \cdot c$: $a \cdot c = 0$

$= (b + a) \cdot (b + c)$: Distributive law

$= (a + b) \cdot (b + c)$: Commutative law

$= 1 \cdot (b + c)$: $a + b = 1$

$= (a + c) \cdot (b + c)$: $a + c = 1$

$= (c + a) \cdot (c + b)$: Commutative law

$= c + a \cdot b$: Distributive law

$= c + 0$: $a \cdot b = 0$

$= c$

$b = c,$ Hence complement is unique.

Theorem 2 (Idempotent law) : If a be an element of a boolean algebra $(B, +, \cdot, ')$ then prove that

(i) $a + a = a$ (ii) $a \cdot a = a$

Proof: (i) Taking L.H.S. $= a + a$

$= (a + a) \cdot 1$: $a.1 = a$

$= (a + a) \cdot (a + a')$: $a + a' = 1$

$= a + a \cdot a'$: Distributive law

$= a + 0$: $a \cdot a' = 0$

$= a$ = R.H.S. : $a + 0 = a$

(ii) Taking L.H.S. $= a \cdot a$

$= (a \cdot a) + 0$: $a + 0 = a$

$= a \cdot a + a \cdot a'$: $a \cdot a' = 0$

$= a \cdot (a + a')$: Distributive law

$= a \cdot 1$: $a + a' = 1$

$= a$ = R.H.S. : $a.1 = a$

Theorem 3: For every element a in a boolean algebra $(B, +, \cdot, ')$ then prove that

(i) $a + 1 = 1$ (ii) $a \cdot 0 = 0$

Proof: (i) Taking L.H.S. $= a + 1$

$= (a + 1) \cdot 1$: $a \cdot 1 = a$

$= (a + 1) \cdot (a + a')$: $a + a' = 1$

$= a + 1 \cdot a'$: Distributive law

$= a + a'$: $1 \cdot a = a$

$= 1$ = R.H.S. : $a + a' = 1$

(ii) Taking L.H.S. $= a \cdot 0$

$= a \cdot 0 + 0$: $a + 0 = a$

$= a \cdot 0 + a \cdot a'$: $a \cdot a' = 0$

$= a \cdot (0 + a')$: Distributive law

$= a \cdot a'$: $0 + a = a$

$= 0$ = R.H.S. : $a.a' = 0$

Theorem 4 : For each pair of elements a and b in a boolean algebra $(B, +, \cdot, ')$ then prove that

(i) $a \cdot ab = a$ (ii) $a \cdot (a + b) = a$

Proof : (i) Taking L.H.S. $= a + a \cdot b$

$= a \cdot 1 + a \cdot b$: $a \cdot 1 = 1$

$= a \cdot (1 + b)$: Distributive law

$= a \cdot (b + 1)$: Commutative law

$= a \cdot 1$: 1 is multiplicative identity

$= a$ = R.H.S. : $a \cdot 1 = a$

(ii) Taking L.H.S. $= a \cdot (a + b)$

$= (a + 0) \cdot (a + b$: $a + 0 = a$

$= a + (0 \cdot b)$: Distributive law

$= a + (b \cdot 0)$: Commutative law

$= a + 0$: 0 is additive identity

$= a$ = R.H.S. : $a + 0 = a$

Theorem 5: In a boolean algebra $(B, +, \cdot, ')$ then prove that

(i) $0' = 1$ (ii) $1' = 0$ (iii) $(a')' = a$

Proof : (i) Taking L.H.S. $= 0'$

$= 0' + 0$: $a + 0 = a$

$= 1$ = R.H.S. : $a + a' = 1$

(ii) Taking L.H.S. $= 1'$

$= 1' \cdot 1$: $a \cdot 1 = a$

$= 0$ = R.H.S. : $a' \cdot a = 0$

(iii) Let $a \in B$, then by definition of complement

$a + a' = 1,\ a \cdot a' = 0,\ a' + (a')' = 1\ and\ a' \cdot (a')' = 0$

Taking L.H.S. $= (a')'$

$= (a')' \cdot 1$: $a \cdot 1 = a$

$= (a')' \cdot (a + a')$: $a + a' = 1$

$= (a')' \cdot a + (a')' \cdot a'$: Distributive law

$= (a')' \cdot a + 0$: $a' \cdot (a')' = 0$

$= (a')' \cdot a + a \cdot a'$: $a \cdot a' = 0$

$= a \cdot (a')' + a \cdot a'$: Commutative law

$= a \cdot ((a')' + a')$: Distributive law

$= a \cdot 1$: $a' + (a')' = 1$

$= a$ = R.H.S. : $a \cdot 1 = 1$

THANKS

METRIC SPACE

DR ANITA MANDLOI

DEFINITION OF METRIC SPACE

- Let X be a non empty set. A mapping $d:X\times X \rightarrow R$ which maps $X\times X$ into R is said to be a metric if d satisfies the following axioms:

- 1. $d(x,y) = 0 = x=y$ for all $x,y \in X$

- 2. $d(x,y)=d(y,x)$ for all $x,y \in X$

- 3. $d(x,y) \leq d(x,z)+(z,y)$ for all $x,y,z \in X$

Distance between x and y

If d is a metric of X, then the ordered pair(X,d)is called a metric space and d(x,y) is called the distance between x and y.

EXAMPLE OF METRIC SPACE

- $d(x,y)= |x-y|$ for all $x,y \varepsilon R$ when d is a metric on R.
- $d(x,y)=0$ if and only if $|x-y|=0$
- $x-y=0 <=> x=y$

$d(x,y)= |x-y| = |-(x-y)| = |y-x| = d(y,x)$

$d(x,y)= |x-y| = |x-z+z-y| \leq |x-z| + |z-y| = d(x,z)+d(z,y)$

QUASI-METRIC SPACE

- Let X be a non empty set. A function
- $P:X\times X\rightarrow[0,\infty]$,
- Which satisfies the following properties for all $x,y,z\leftrightarrow X$
- $P(x,x)=0$, i.e. $P(x,y)=0$ if $x=y$
- $P(x,y)=P(y,x)$,
- $P(x,y)\leq P(x,z)+P(z,y)$ is called quasi metric space on X.
- $P(x,y)<\infty$

REAL ANALYSIS
DR. ANITA MANDLOI

NEIGHBOUREHOOD OF A POINT LIMIT POINT OF A SET
B.SC III YEAR

NEIGHBOUREHOOD

- Let (x,d) be a metric space and xεX. A subset N of X is said to be a neighbourhood of x if there exist an open set G such that
- $x \in G \subseteq N$
- In case N is an open set it is called an open neighborhood.

- Since G is open, there exist r greater than zero such that
- $S_r(x)$is a subset of G is a subset of N. Then N be a nbd of x if and only if there exist an open sphere centered at x and contained in N.
- Since every open sphere is an open set, it follows that every open sphere contained in x is an open nbd of x.
- An open sphere centered at x is an open nbd of x.

- Let (X,d) be a metric space and A be a subset of X. A point $p \in X$ is called a limit point if and only if every nbd of p contains a point of A distict from p.
- $p \in X$ is called limit point of A if and only if for each $r>0$,
- $A \cap (N_r(p)-\{p\}) \neq \Phi$

RIEMANN INTEGRAL
B.SC III YEAR

DR. ANITA MANDLOI

PARTITION

- Let I=[a,b] be a given closed and bounded interval in R. Subdivided this interval into n sub-intervals by means of points x_1, x_2 ,x_3,….xn choose arbitrary. Then a partition of I is a finite ordered set P of points in I as defined-

$P = \{ x_o, x_1, x_2, x_3, \ldots . xn\}$

Where $a= x_o \leq x_1 \leq x_2 \leq x_3 \leq \ldots \leq xn=b$

NORM

- Let P= { x_0, x_1,x_2,x_3,….xn} be any partition of

  [a,b] then the greatest of the lengths of

  {Δx_i: i=1,2,3,……n}

REFINEMENT

Let P= { x_0, x_1,x_2,x_3,....xn} and

P*= {y_o y_1,y_2,... yn} be a partitions of I=[a,b]. We say that P*is a

refinement of P if and only if each partition point $x_i \varepsilon P \Rightarrow x_i \varepsilon$

P*for all i.

Govt. Dr Shyama Prasad Mukherjee Science and Commerce College

Dr. Naval Singh

Normed Algebra and Banach Algebra and Theorem

A **Normed algebra** is a normed space which is an algebra such that

$$\|xy\| \leq \|x\|\|y\| \ \forall x, y \in A$$

And if A has an identity e, then

$$\|e\| = 1$$

A **Banach algebra** is a normed algebra which is complete, considered as normal space.

Theorem:-

Let A be a Banach algebra with identity e. If $x \in A$ satisfies $\|x\| < 1$ then e-x is invertible, and

$$(e-x)^{-1}=e+\sum x^{j} \qquad (1)$$

Proof:-

We know that, $\|x^j\| <= \|x\|^j$, so that $\sum\|x^j\|$ converges since $\|x\|<1$. Hence the series in (1) converges absolutely, so that it converges because A is complete.

Let s denotes its sum, and let us show that $s = (e-x)^{-1}$. Now,

$$(e-x)(e+x+\ldots\ldots+x^n)$$

$$= (e+x+\ldots\ldots+x^n)-(x+\ldots\ldots+x^n+x^{(n+1)})$$

$$= e-x^{(n+1)} \qquad (2)$$

Now let $n \to \infty$. Then $x^{(n+1)} \to 0$ since $\|x\|<1$, and (2) implies

$$(e-x)s = s(e-x) = e$$

Since multiplication in A is continuous. Hence $s = (e-x)^{-1}$ and (1) holds.

Operations Research

DR.RAJESH SHRIVASTAVA

MATHEMATICS

GOVT.SHYMA PRASAD MUKHRJEE COLLEGE , BHOPAL

Operation Research and its scope:

- Origin and Development of operations research, Characteristics of Operation Research.
- Model in Operations Research, Phase of Operations Research, Uses and Limitations of Operations Research,
- Linear Programming Problems: Mathematical formulation, Graphical solution method.
- General Linear Programming Problem: Simplex method, exceptional cases, artificial variable technique, Big-M method, Two-phase method and cyclic problem, problem of degeneracy.
- Duality, fundamental properties of duality and theorem of duality .

Origin of Operations Research

- Operations research was first introduced in 1940vby McClosky and Trefthen in Boudsey U.K.
- During world war II military management called on scientists from various disciplines and organized them to improve the execution of various military projects.

Origin of Operations Research

- They suggested certain approaches that showed remarkable progress.
- This new approach to systematic and scientific study of the operations of the system was called the "operations research or operational research" abbreviated as O.R.

Origin of Operations Research

- Following the world war II, the success of military teams attracted the attention of industrial managers were seeking solutions to their complex executive-type problems.
- During the year 1950,O.R. achieved recognition as a subject worthy of academic study in the universities.

Origin of Operations Research

- Since then the subject has been gaining more and more importance for the students of Economics, Management, Public Administration, Behavioral Sciences, Social work, Mathematics , Commerce and Engineering.

Origin of Operations Research

- The ‘International Federation of Operations Research Societies was established in 1957.
- In India Operations Research come into existence in 1949 as an O.R. unit in Regional Research Laboratory at Hyderabad.

Origin of Operations Research

- The ‘International Federation of Operations Research Societies was established in 1957.
- In 1953 an Operations Research unit was established in the Indian Statistical Institute Calcutta for national planning and survey.

Origin of Operations Research

- The Operations Research Society of India was formed in 1957 and it become member of the International Federation of O.R. Societies in 1959.
- Prof. Mahalonobis made the first important application of Operations Research in second Five Year Plan of India.

Origin of operations research

It was first used by army to during second world war.

They are used to search for enemy submarine, supply food, how to win the war.

History of Operations Research

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History of Operations Research

- The effective use in military spread it in the government department and industries.

History of Operations Research

- So many OR professionals are being used in different places.
- Since 1940 it is a effective method of decision making.

Definitions :

1. Operations research is an art of giving bad answers to the problems which otherwise have worse answers.

   - T.L.Saaty

2. O.R. is a scientific approach to problem solving for executive management.

   - Winsten

Definitions :

3. Operations research is a scientific knowledge through interdisciplinary team effort for the purpose of determining the best utilization of limited resources. - H.A.Taha

4. Operations research is an art of winning war without actually fighting them. - Clark

Definitions :

5. A scientific approach to decision making, which seeks to determine how to design and operate the system. Usually under condition requiring allocation of resources. - **Winsten**

Features of O.R. :

1. Decision Making
2. Scientific Approach
3. Objective
4. Inter-disciplinary Team Approach
5. Digital Computers

Operations Research

Dr. RAJESH SHRIVASTAVA

Dept. of Mathematics,

GOVT. SHYMA PRASAD MUKJRJEE COLLEGE ,

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Characteristics of O.R. :

1. Decision Making
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Modeling in O.R. :

1. Definition of model in OR
2. Characteristics of good Model
3. Limitations of good Model
4. Classification of Models
5. General solution methods for OR models

Definition :

1. A model in Operations research is a simplified representation of an operation or a process in which only the basic aspects or the most important features of a typical problem under consideration are considered.

Objective of a model

1. To provide a means for analyzing the behavior of the system
2. Improving the performance.

Characteristics of a Model :

1. New formulation without change in frame
2. Small assumptions
3. Simple and coherent
4. Less number of variables
5. Open to parametric type of treatment
6. Minimum time consumption

Limitations of a Model :

1. An attempt in understanding operations
2. Not absolute in any sense
3. Validity by verification through only experiment

Classification of Models :

1. Models by degree of abstraction
2. Models by function
3. Models by structure
4. Models by nature of the environment
5. Models by extent of generality

Classification of Models :

1. Models by degree of abstraction

   (i) Language model

   (ii) Case study model

Classification of Models :

2. Models by function

(i) Descriptive models

(ii) Predictive models

(iii) Normative models

Classification of Models :

3. Models by structure

(i) Iconic or Physical models

(ii) Analogue models

(iii) Symbolic or mathematical models

Classification of Models :

3(iii) Symbolic or Mathematical Models

- **(a)** Inventory Model for 4M
- **(b)** Allocation Model
- **(c)** Sequencing Model
- **(d)** Routing Model
- **(e)** Queuing Model
- **(f)** Decision Model
- **(g)** Replacement Model
- **(h)** Heuristic Model

Classification of Models :

4. Models by nature of the environment

   (i) Deterministic models

   (ii) Probabilistic or stochastic models

Classification of Models :

5. Models by the extent of generality

   (i) Specific models

   (ii) General models

Solution method for OR Models :

1. Analytical or Deductive Methods
2. Numerical Methods
3. Monte Carlo Method
4. Models by nature of the environment
5. Models by extent of generality

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Phases of O.R. :

1. Judgment Phase
2. Research Phase
3. Action Phase

Methodology of OR:

1. Formulation of the problem
2. Construction of a mathematical model
3. Deriving the solution from the model
4. Validity of the model
5. Establishing the control over the solution
6. Implementation of the final result

Uses or Applications of OR:

1. Finance, Budgeting and Investment
2. Marketing
3. Physical Distribution
4. Purchasing, Procurement and Exploration
5. Personnel and Production
6. Research and Development

Uses of Operations Research

- Industries
- Defense
- Agriculture
- Public utility
- Airlines
- Railways

Opportunities of OR:

1. Determination of solution
2. Visualizing the objective and constraints
3. Better decision for organization
4. Minimization of time consumption
5. Better optimal solution without causing any serious damage in system
6. Provide tool for scientific analysis.

Opportunities of OR:

7. Provide solutions for business problems
8. Future predictions can be specified by operations research.
9. Give power to our decision making process
10. Better control , better decisions, better co-ordination.

Limitations of OR:

1. Complex problems some times have expensive solution by OR
2. Dose not represent real problem
3. Frequent change in data becomes costly in model construction
4. Complex Model and computer
5. Non-quantifiable and psychological factors are not considered.

Limitations of OR:

6. It is tool of analysis not complete decision making process
7. Internal resistance
8. Sometimes analytical skills of people are different so it takes time to reach to any decision.
9. Do not consider qualitative or Emotional factors which are very important

Questions

Qu.1: What is Operation Research. Explain its scope.

Qu.2: Discuss Origin and Development of operations research and explain the Characteristics of Operation Research.

Qu.3: Define different Models and Phases or scientific approaches in Operations Research.

Qu.4: Explain Uses and Limitations of Operations Research.

Qu.5: "Operation research is a bunch of mathematical techniques." Comment.

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Characteristics of O.R.

- Operation research can be consider as a source to other new sciences.
- The word Operation may be defined as some action that we apply to some problem or hypothesis.
- The word research is an organized process of seeking out facts about the problem or hypothesis.

Characteristics of O.R.

- Operation research may be describe as the “Science of use” or “Quantitative common sense”.
- Operation research may be describe as the “Scientific approach to decision-making problems”.

Characteristics of O.R. :

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2. Scientific Approach
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